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Strong coupling and entanglement of two coupled quantum dots

Elena del Valle
School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, United Kingdom

Abstract. I present a theoretical study of the strong and weak coupling of two quantum dots, considered as two-level systems. Under an incoherent continuous excitation, new regimes are found where the effective coupling is reinforced, causing the standard dressed states and Rabi doublet to split in four. In this regime, entanglement is enhanced above the previously reported values for thermal excitation. Finally, I consider the case where the dots are coupled through another mode: a cavity mode (harmonic oscillator) or a third two-level system.

Introduction

Strong light matter coupling with self-assembled quantum dots in microcavities has been achieved by many groups. Antibunching in the dot emission has been observed [1; 2], evidencing its quantum nature and opening a broad range of quantum applications for these semiconductor devices, such as efficient single and two photon sources. Quantum dots, as two-level systems (or qubits), have the advantage of a solid state system, such as the possibility of optical and electrical manipulation, scalable implementations, etc. Disadvantages, as compared to atomic transitions or superconducting qubits, are, for instance, the high degree of dephasing caused by interaction with the environment [3]. Another specificity of typical experiments with quantum dots that has proved to be more relevant [4; 5], is the incoherent continuous excitation. In the present model, I include both elements together with dissipation, in order to fully account for the incoherent processes that may hinder, but also, surprisingly, enhance the coherent coupling.

The study of the coupling and entanglement of two quantum dots (or two-level systems), subject to decoherence, is interesting for many reasons. The system represents an analytically solvable example of entanglement in an open quantum system. Regarded as a fundamental problem, it has been theoretically investigated in the past, but only in the presence of thermal noise [6–9]. Moreover, with the rapid progress of growth and control of quantum dots in microcavities, the possibility of coupling strongly two quantum dots has become a reality. Recently, Laucht *et al.* have observed clear signatures of two dots coupling with and through a cavity mode [10]. Hints of inter-dot tunneling have also been reported previously [11].

1. Theoretical description

The quantum dots are considered as two-level systems with lowering operators σ_i ($i = 1, 2$), frequencies ω_i and coupled with strength g. They may also couple to a third intermediate mode σ_3 that can be fermionic (another quantum dot) or bosonic (a cavity mode $\sigma_3 \to a$) and provides the reference energy ($\omega_3 \equiv 0$). The Hamiltonian reads:

$$H = \sum_{i=1}^{2} [\omega_i \sigma_i^\dagger \sigma_i + g_i(\sigma_3^\dagger \sigma_i + \sigma_i^\dagger \sigma_3)] + g(\sigma_1^\dagger \sigma_2 + \sigma_2^\dagger \sigma_1)\,. \quad (1)$$

The total density matrix matrix of the system, $\tilde\rho$, follows a master equation including all incoherent processes [4; 12]:

$$\partial_t \tilde\rho = i[\tilde\rho, H] + \sum_{i=1}^{3} \Big[\frac{\gamma_i}{2}\mathcal{L}_{\sigma_i} + \frac{P_i}{2}\mathcal{L}_{\sigma_i^\dagger} + \frac{\delta_i}{2}\mathcal{L}_{\sigma_i^\dagger\sigma_i}\Big]\tilde\rho\,. \quad (2)$$

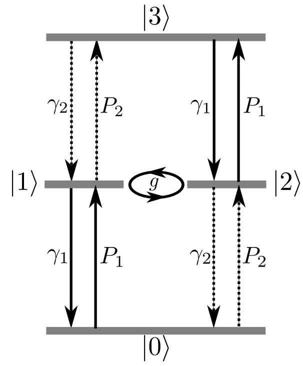

Fig. 1. Level scheme and parameters of the system under study: two coupled two-level systems with pump and decay.

The parameters γ_i, P_i and δ_i are the rates of decay, pumping and pure dephasing of the modes, which appear in the equation as Lindblad terms ($\mathcal{L}_O\tilde\rho \equiv 2O\tilde\rho O^\dagger - O^\dagger O\tilde\rho - \tilde\rho O^\dagger O$). I investigate the steady state of the bipartite system, ρ, by solving exactly the equation $\partial_t\tilde\rho = 0$ and tracing over the intermediate mode, if present. The steady state is a mixture of all the four possible states, sketched in Fig. 1.

The strong coupling regime, determined by the appearance of dressed states, can be investigated by means of the *photoluminescence* or *power spectrum* of one of the dots, $S_1 \propto \Re\int_0^\infty \langle\sigma_1^\dagger(t)\sigma_1(t+\tau)\rangle e^{i\omega\tau}d\tau$. The required two-time correlator follows from the *quantum regression formula* [13]. The degree of entanglement in a mixture of two quantum dot states is given by the *concurrence* [14], C, equal to 0 for completely separable states and 1 for an entangled pure state. Closely related is the degree of purity in the mixture, given by the *linear entropy* [15], S_L, equal to 0 for a pure state and 1 for a maximally mixed state.

2. Strong coupling of two quantum dots

The problem of two directly coupled quantum dots with pump and decay can be fully solved analytically [16]. This allows for a detailed analysis of the physics of strong and weak coupling and the different dressed states arising in the system. For configurations with dissipative environments ($\gamma_i > P_i$), which include the thermal baths, or gain environments ($P_i > \gamma_i$), the strong coupling is *standard*, featuring only two dressed states. More importantly, in the case where one quantum dot is in contact with a gain medium ($P_1 > \gamma_1$), and the other with a dissipative medium ($\gamma_2 > P_2$), new coupling regimes arise, that I call *second order strong coupling* (SSC) and *mixed coupling* (MC). They are characterized by the appearance of four—instead of two—dressed

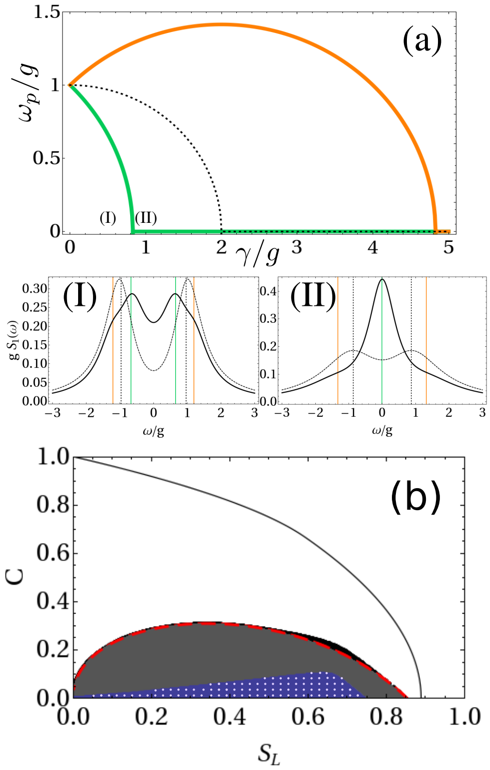

Fig. 2. (a) Double splitting (positive part only) of two coupled quantum dots (solid) and standard Rabi splitting of a four-level system (dashed), in the optimally pumped configuration. The resulting spectra of emission is a quadruplet (I) or triplet (II) instead of the standard Rabi doublet (in dashed). (b) Distribution of C and S_L for all possible two quantum dot configurations (shaded region). The thin line corresponds to the maximum concurrence for a given linear entropy in a general bipartite system. The thick dashed line corresponds to the optimally pumped configuration. In dotted, the particular case of thermal reservoirs [8].

states, that undergo a double anticrossing. This is shown in Fig. 2(a) (at positive energies) for the optimally pumped case: $\gamma_1 = P_2 = 0$, $\gamma_2 = P_1 = \gamma$. In SSC, the spectrum of emission (I) has a quadruplet structure (with peaks positioned on the vertical solid lines) and in MC, a triplet one (II) due to the saturation of one pair of dressed states. This is to be compared with the standard results of a four level system (such as a single quantum dot with a biexciton state), superimposed in dashed. The reason for this exotic coupling regimes is to be found in the enhancement of coherence induced by the links that pump and decay establish between diagonal transitions $|0\rangle\langle i|$ and $|j\rangle\langle 3|$ $(i \neq j)$. A coherence circulation loop appears in the system that accompanies and complements the Rabi oscillations between $|1\rangle \leftrightarrow |2\rangle$, producing new dressed states.

It is remarkable that the optimally pumped configuration provides also the maximum entanglement available in this system [17]. This is shown in Fig. 2(b), where the region that our system can access in the concurrence-linear entropy plane is shaded in grey/black. The optimally pumped configuration (dashed thick line) encloses most of this region, providing in good approximation the maximum entanglement for a given linear entropy. In any case, it also includes the absolute maximum entanglement, $C_{\max} = (\sqrt{5}-1)/4 \approx 0.31$, at $\gamma_{\max}/g = 1+\sqrt{5}$. These high values of entanglement are possible thanks to the inversion of population in the driven quantum dot, which reduces S_L while the coherent coupling is still effective between the dots. This should be compared with previous results [8; 9] with thermal noise at resonance. I find that higher values of entanglement can be still obtained in those cases by allowing some detuning between the quantum dots (dotted region).

3. Coupling mediated by a third mode

Theoretically, the simplest element to mediate the coupling between two quantum dots is another two-level system (a resonator or a third quantum dot). Analytical solutions can still be found and directly compared with the direct coupling results. I find that entanglement is completely lost in the optimally pumped configuration although some concurrence can be obtained by having the mediator pumped and the two dots only decaying [17]. Following a different scheme, entanglement is also possible by forcing the system into an entangled state, such as the singlet (Bell) state $|1\rangle - |2\rangle$. This can be achieved if the two quantum dots are fed by the same pumping reservoirs [18].

A second possibility is that the coupling between dots is mediated by a cavity mode (harmonic oscillator). Analytical results are not available in general, and Eq. (2) must be numerically solved to obtain entanglement and emission properties. The same scheme of entanglement by the effect of a common dot reservoir is still feasible [19]. From another point of view, the cavity emission is largely modified by the strong coupling with the dots, increasing its lasing efficiency [19] or even leading to two-photon lasing [20].

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