"*Computación I*"
Projects

Fabrice P. Laussy
fabrice.laussy@gmail.com — http://laussy.org/wiki/Computacion_I

February 22, 2016

1 Random number generators

The generation of random numbers is an important problem in several areas of computation. We will study various approaches to generate random numbers that follow arbitrary distributions. We will focus on uniform distributions of integers as far as generation is concerned and then turn to the "*inverse transform sampling*" method for distributing them accordingly. In each case, we will study the quality of "randomness" of our generators. We will explore simple methods such as "*middle-square method*", "*linear congruential generators*" and others ranging from fairly elementary to, time allowing, state-of-the-art.

2 Celullar automata

A cellular automaton is a set of rules applied iteratively on a finite grid, not as a numerical approximation of a field, but as an exact universe by itself. In this project, we will study the behaviour of so-called "*elementary cellular automata*" which are 1D and with two only possible states. Each cell's state is determined in time by its previous state and that of its two neighbors. We will explore such notions as complexity and universality. Then we will consider cells that can take more than two values and, in particular, consider "*cyclic cellular automata*" that exhibit behaviors of interacting particles. Time allowing, we will also study such celullar automata in 2D.

3 Recurrence relations

Recurrence relations are equations that recursively define a sequence of numbers according to simple rules, such as the Fibonacci sequence defined by $F_n = F_{n-1} + F_{n-2}$ with $F_1 = F_2 = 1$. We will first explore a variation known as "*lagged Fibonacci generators*", $F_n = F_{n-j} \star F_{n-k} \pmod{m}$ (with $\star$ an operation to be determined), that are popular to generate pseudo-random numbers. Then, we will explore the nonlinear relation $x_{n+1} = rx_n(1 - x_n)$ for $0 \leq x \leq 1$ and $0 < r \leq 4$, the so-called "*logistic map*", that exhibits complex patterns (chaotic behavior).

4 Quantum computation

Quantum computation is a paradigm of computation that includes the laws of physics, in particular, quantum physics, which comes with limitations but also extensions as compared to the platonic Boolean paradigm. We will simulate a quantum computer classically, and thus explore what would be the gain of such a device if it could be implemented in a laboratory. We will start from elementary algorithms such as "*boson sampling*", simulating simple "*quantum gates*" and, time allowing, implement more complex schemes such as the "*Deutsch–Jozsa algorithm*" or even tackle the star of quantum computation: "*Shor's algorithm*".

5 Fractals in $\mathbb{C}$

Iterations of the function $f(z) = z^2 + c$ of the complex number z (starting from $z = 0$) lead to a sequence of numbers that diverges or not. The set of points that remain bounded yields a peculiar structure, a so-called "*fractal set*" that is self-similar. We will explore this structure and some variations of it numerically. Time allowing, we will play with "*arbitrary precision arithmetic*" to go beyond machine precision and access as a remote corner as we can of the "*Mandelbrot set*".

6 Volterra-Lotka equations

We will study the following coupled nonlinear first-order set of differential equations (α, β, γ and δ being real parameters):

$$\frac{dx}{dt} = \alpha x - \beta xy\,, \tag{1a}$$

$$\frac{dy}{dt} = \delta xy - \gamma y\,, \tag{1b}$$

known as the *"Volterra-Lotka" equations*, that describe predator-prey interacting systems. We will stuby both the problem of "*numerical stability*" of various algorithms to integrate these equations and also study the dynamical stability of the trajectories of this dynamical system in the context of "*Stability theory*". Time allowing, we will turn to more complicated dynamical systems such as the "*Belousov-Zhabotinsky oscillations*".

7 Robust measures of tail weight

Outliers can come to dominate completely the behaviour of a system, a phenomenon dramatized by so-called "*Black Swan theory*". It is an important problem to characterize from empirical statistical data the behaviour of the tails of the distribution of random events, in particular to measure the impact of "*fat tailed distributions*". We will study various proposed estimators and compare them to numerical experiments. In particular, we will investigate the distributions of the so-called "*kurtosis*", for sums of random variables, e.g., $\sum_{i=0}^{N} X_i$ for N Normally distributed random Variables X_i.

8 Statistical hypothesis testing

As much as descriptive statistics is simple and intuitive, so-called "*inferential statistics*" is subtle, complex and far-reaching. We will explore inferential statistics with numerical experiments, such as "*Fisher's exact test*" and his "*Lady Tasting Tea*" application. In particular, we will study how the bias in the estimation of the population variance is affected by "*Bessel's correction*". We will also get acquainted with less less popular distributions and in this process identify "*univariate distributions relationships*".

***

First problem is to distribute these projects between the groups so that every project be assigned and maximizing people's choice preferences (to be decided algorithmically):

1. Group Beatriz Herrero Badorrey — Fco. Javier Castillo
2. Group Pablo Bastante Flores — Fernando Chacua Sá nchez
3. Group Maria Pillar Bueno Rodríguez — Nadia González
4. Group David Barba Gonzlez — Félix Ayllón Muñoz
5. Group Juan José García Esteban — Raú l Espinosa Copeda — Juan Ramón Deop
6. Group Sergio Domínguez Vidales — Alberto Castellano Mora
7. Group Celia de Santos Pedrazuela — David Caldevilla Asenjo
8. Group Manuel Calvo