Exercises to be handed in on 2 April 2013. Please label your work with your name and surnames as well as DNI.

1. Take the series $\sum_{n=1}^{\infty} z/n$. Visualise on the complex plane the n-th partial sums, s_n, for the case $z = 2(1+i)/3$, by connecting each pair s_n, s_{n+1} with a segment.

2. Find the radius of convergence of the following series:

   (a)
   $$\sum_{n=0}^{\infty} \frac{z^n}{n!}$$

   (b)
   $$\sum_{n=0}^{\infty} n! z^n$$

   (c)
   $$\sum_{n=0}^{\infty} z^{n!}$$

3. Given the series $\sum_{n=0}^{\infty} a_n z^n$ with radius of convergence R, find the radius of convergence of the following series:
   $$\sum_{n=0}^{\infty} a_n^2 z^n$$

4. Study the convergence and then find the sum of the following series:
   $$\sum_{n=0}^{\infty} \frac{(z+i)^n}{(1+i)^{n+1}}$$

5. Find the power series that has as sum the following function
   $$f(z) = \frac{1}{z^2+z+1}$$
   Note that $(1-z^3) = (1-z)(z^2+z+1)$. Also find the radius of convergence.

6. Suppose that the power series $\sum_{n=0}^{\infty} a_n z^n$ has a recurring sequence of coefficients $a_n = a_{n+k}$, with k a fixed positive integer. Prove that the series converges for $|z| < 1$ to a rational function $p(z)/q(z)$, $p(z)$ and $q(z)$ being two polynomials, and that the roots of $q(z)$ are all on the unit circle.

7. Given
   $$\frac{1}{1-z-z^2} = \sum_{n=0}^{\infty} F_n z^n$$
   show that: $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$.