Universal signatures of lasing in the strong coupling regime

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ABSTRACT

An almost ideal thresholdless laser can be realized in the strong-coupling regime of the light-matter interaction, with Poissonian fluctuations of the field at all pumping powers. Here, we show that this ideal scenario is thwarted by quantum nonlinearities when crossing from the linear to the stimulated emission regime. A universal jump in the normalized intensity correlation function is predicted to occur, the measurement of which could be used to establish a standard fingerprint of the onset of lasing in the strong coupling regime.

Keywords: Microcavity, cavity QED, one-atom laser

1. INTRODUCTION

"Lasing", in its modern understanding, is associated to quantum coherence rather than to the historical mechanism of stimulated emission overcoming spontaneous emission. This is illustrated by atom lasers[1] or polariton lasers.[2] Although most lasers find their applications in their high intensity and/or highly directed beam, from a fundamental point of view, Glauber's definition of coherence as autocorrelation functions of the field $N_a[n] = \langle a^{\dagger n} a^n \rangle$ that factor out as $\langle a^\dagger a \rangle^n$ provides the best formal definition of lasing.[3] A single emitter can be used to generate such a coherent field, provided that it is strongly coupled to it,[4] in which case the interaction is reversible and, thus, can lead to a coherent enhancement of the field intensity by piling up quanta through Rabi oscillations. In contrast, conventional lasers typically operate in the *weak-coupling regime*, where the interaction is perturbative and coherence is weak until stimulated emission sets in. This calls for a large number $N \gg 1$ of emitters to generate a sizable field intensity. The inversion of the population between the quantum states of the isolated emitters leads to a pumping threshold. With a single emitter, if the spontaneous emission rate into modes other than the cavity is small, the growth in the population of photons appears to exhibit no threshold.[5] Here, we consider the coherence properties of the single-emitter laser as well and show how—rather than a small spontaneous emission of the emitter into other modes—the configuration that optimises strong coupling is the one that most closely realizes a thresholdless laser. Coherence created at high pumping by stimulated emission can also be sustained at vanishing pumping through strong coupling with identical decay rates of the light-matter components. Most strikingly, we predict a universal "jump" when moving between these two limits. This prevents the realization of an ideal thresholdless single-emitter laser producing a coherent field at all pump levels, but enables a fundamental characterisation of the device.

2. STATE OF THE ART

Experimentally, one can implement a single-emitter laser with an individual trapped atom in an optical cavity.[6] This is the historical realization, which gave it its more common denomination of "one-atom laser". One can also use *artificial atoms*, such as a superconducting qubit in a superconducting transmission line resonator[7] or a quantum dot in a semiconductor microcavity.[8,9] Signatures of strong coupling at the level of two excitations have been reported in each of these systems,[6,10,11] as well as "one-atom lasing",[12–14] showing that they can be considered as quasi two-level quantum systems. Theoretically, a significant body of work has addressed the steady state properties of the single-emitter laser.[4,15–22,24,25] It is described, at its most fundamental level, by

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the coupling between a two-level system σ and a cavity mode a in a dissipative environment. This leads to a master equation:

$$\partial_t \rho = -i[H_{\mathrm{JC}}, \rho] + \{\frac{\gamma_a}{2}\mathcal{L}_a + \frac{\gamma_\sigma}{2}\mathcal{L}_\sigma + \frac{P_\sigma}{2}\mathcal{L}_{\sigma^\dagger}\}\rho \tag{1}$$

(in units of $\hbar = 1$), where $\mathcal{L}_c\rho = (2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$ is the Lindblad term associated with decay of the cavity (γ_a) or the emitter (γ_σ), its pumping (P_σ) and H_{JC} is the celebrated Jaynes–Cummings Hamiltonian,[23] at the heart of the quantum dynamics: $H_{\mathrm{JC}} = g(\sigma^\dagger a + a^\dagger \sigma)$. The steady state can be expressed completely in terms of the photon correlators, which obey the equations:[18]

$$\Big[1+\frac{\Gamma_\sigma + (2n-1)\gamma_a}{\kappa_\sigma} + \frac{n\gamma_a}{\Gamma_\sigma + (n-1)\gamma_a} - \frac{2P_\sigma}{\Gamma_\sigma + n\gamma_a}\Big] N_a[n] = \frac{nP_\sigma}{\Gamma_\sigma + (n-1)\gamma_a} N_a[n-1] - \frac{2\gamma_a}{\Gamma_\sigma + n\gamma_a} N_a[n+1]\,, \tag{2}$$

where we have introduced $\Gamma_\sigma = \gamma_\sigma + P_\sigma$ and $\kappa_\sigma = 4g^2/\gamma_a$, the Purcell rate of transfer of population from emitter to the cavity mode. The main observables of interest are the cavity population, $n_a = N_a[1]$, directly linked to the intensity emitted by the device through $I = \gamma_a n_a$, and the normalized nth-order autocorrelation function $g^{(n)} = N_a[n]/n_a^n$, especially the second order one, $g^{(2)}$, measured by probing the photon temporal statistics at zero time delay. The probability of the emitter being in the excited state $n_\sigma = \langle \sigma^\dagger \sigma \rangle$ is a dependent variable ($n_\sigma = (P_\sigma - \gamma_a n_a)/\Gamma_\sigma$), which we thus do not need to consider any further.

3. "SINGLE-PHOTON" AND "STIMULATED EMISSION" LASING

The set of equations (2) can be solved analytically to very good approximation.[24] Two specific cases, when the field intensity scales linearly with pumping, are of interest for our present discussion. Calling the rate of growth C_i, i.e.,

$$n_a = C_i P_\sigma\,, \tag{3}$$

with $i = 1, 2$, one can show[24] that, on the one hand, the system is in a "linear" regime where only the first rung of the Jaynes–Cummings ladder is occupied, and for which:

$$C_1 \approx \frac{\kappa_\sigma}{\kappa_\sigma + \gamma_\sigma}\frac{1}{\gamma_a + \gamma_\sigma}\,. \tag{4}$$

This is shown in Fig. 1(a) for weak pumping level, where n_a follows the dashed straight lines, given by Eq. (4). On the other hand, at higher pumping level, the field intensity also scales linearly with pumping, but this time with a rate independent of γ_σ, which is the situation where stimulated processes completely dominate spontaneous decay:

$$C_2 \approx \frac{1}{2\gamma_a}\,. \tag{5}$$

This is shown in Fig. 1(a) by the point where all lines converge (since γ_a is fixed). There is therefore a "jump" $\mathcal{J}$ between the two rates of efficiency in the transition from the linear to the lasing regime:

$$\mathcal{J} = \ln(C_2/C_1) \approx \ln(\gamma_a + \gamma_\sigma) - \ln(2\gamma_a)\,. \tag{6}$$

All these results become exact in the limit $\kappa_\sigma \gg \gamma_\sigma$. This jump changes sign when $\gamma_\sigma = \gamma_a$, which is the condition that maximises the strong-coupling criterion for any given coupling strength:

$$4 > |\gamma_a - \gamma_\sigma|/g\,. \tag{7}$$

Cases that satisfy Eq. (7) with $\gamma_\sigma < \gamma_a$ result in a reduction of the efficiency of pumping when crossing from the linear to the quantum regime, while cases $\gamma_\sigma > \gamma_a$ undergo an enhancement, as stimulated emission overcomes spontaneous emission according to the conventional lasing scenario. The reduction is perhaps more surprising. It is maximum when $\gamma_\sigma = 0$ (no spontaneous emission) in which case $\mathcal{J} = \ln(1/2)$, the factor $1/2$ being linked to the inversion of population (in the lasing region, $n_\sigma = 1/2$).

In addition to linear population increase, the ideal thresholdless laser should also have perfect coherence at all pumping powers. In the sense of Glauber, this means a stable light source with a pinned Poissonian

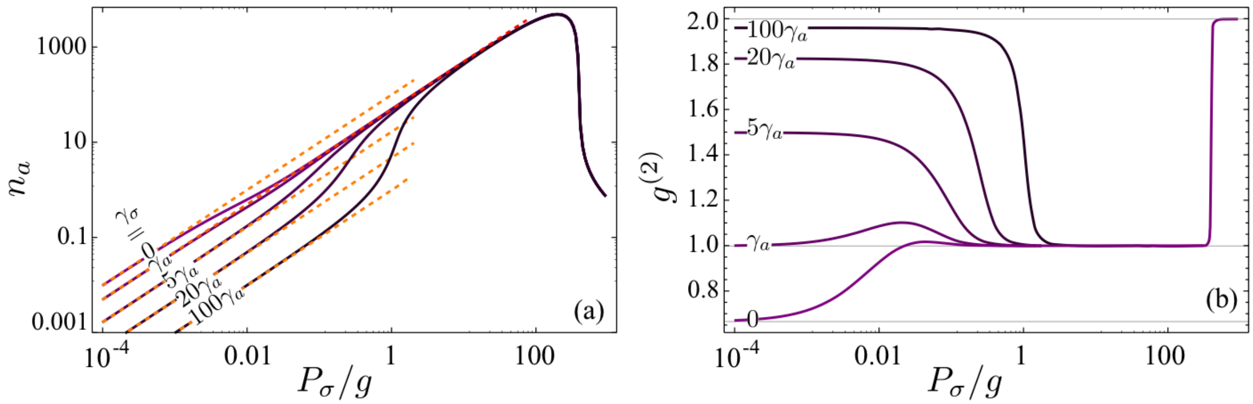

Figure 1. (Colour online) (a) Cavity population n_a and (b) second order correlation $g^{(2)}$ for various γ_σ (with $\gamma_a/g = 10^{-2}$). (a) can be considered to be analogous to the input–output characteristics of the system. The linear variation $n_a = C_1 P_\sigma$ is superimposed on the dashed orange lines. In the stimulated emission regime, all lines converge to the same one (red). The second uppermost curve where $C_1 = C_2$, that covers both regimes, is the closest approximation to an ideal thresholdless laser in strong-coupling. It however deviates slightly in the intermediate region. This deviation becomes compelling in $g^{(2)}$, where it arises as a bunching of photons when turning a perfect Poissonian distribution at small pump into another one at high pump. This universal curve in rescaled units is magnified in Fig. 2.

fluctuation of its statistics at all intensities, even those much below unity. This is known to be the case when stimulated emission dominates.[4] We now consider the case of vanishing pumping and small intensities to see whether coherence can be sustained in this regime too.

Since photon correlators follow $N_a[n] \propto P_\sigma^n$ at vanishing pump, a finite value for all $g^{(n)}$ is assured independently of the truncation scheme used to solve Eq. (2). Thus, we obtain the exact expression for the general correlation function in the limit $P_\sigma \to 0$:

$$g^{(n)}_{P_\sigma\to 0} = n g^{(n-1)}_{P_\sigma\to 0} \frac{\kappa_\sigma + \gamma_\sigma}{\kappa_\sigma + \gamma_\sigma + (n-1)\gamma_a} \frac{\gamma_a + \gamma_\sigma}{(2n-1)\gamma_a + \gamma_\sigma}\,, \tag{8}$$

starting from $g^{(1)}_{P_\sigma\to 0} = 1$. In the very strong coupling regime (where κ_σ is the largest parameter) the second-order correlator reads:

$$g^{(2)}_{P_\sigma\to 0} \approx \frac{2(\gamma_a + \gamma_\sigma)}{3\gamma_a + \gamma_\sigma}\,, \tag{9}$$

which is always between 2/3 and 2, as shown in Fig. 1(b). This result has also been recently obtained by a continuous fraction expansion.[25] The sought condition $g^{(2)}_{P_\sigma\to 0} = 1$ is, again, $\gamma_\sigma = \gamma_a$, the same criterion as the one that aligns the two linear regimes in the input–output characteristics. All higher order correlators, from Eq. (8), also satisfy $g^{(n)}_{P_\sigma\to 0} = 1$ in this case, showing that the state is exactly Poissonian or, in the sense of Glauber, perfectly coherent.

The situation of an ideal thresholdless laser would thus seem to be realized when:

$$\gamma_a = \gamma_\sigma\,, \tag{10}$$

namely, the light field would seem to be coherent to all-orders with an intensity that increases linearly throughout the entire excitation scheme, from arbitrarily small values of pumping. Before discussing a crucial limitation in this scenario, we revisit the concept of thresholdless lasing and how to quantify it. The threshold of a conventional laser is measured by its β factor, which approaches unity as the threshold reduces, a concept that has been extended to the one-atom laser:[26] $\beta = [\kappa_\sigma/(\kappa_\sigma + \gamma_\sigma)][\gamma_a/(\gamma_a + \gamma_\sigma)]$. In our approximation of $\kappa_\sigma \gg \gamma_\sigma$, β is related to our jump between the linear increases of the single-photon and stimulated emission lasing regimes as $\mathcal{J} = \ln(1/(2\beta))$. The β factor is the fraction of emission in the lasing mode (the cavity), which is stimulated, over other channels of emission, most importantly spontaneous emission which is always present, at least in weak coupling. Strong coupling being this regime where spontaneous emission becomes a reversible process, we argue that the definition $\beta = 1$, or $\mathcal{J} = -\ln(2)$, suits best weak-coupling lasers and that in strong-coupling, $\beta = 1/2$ or $\mathcal{J} = 0$ is the closest, albeit as we will discuss shortly, non-ideal approximation to thresholdess lasing operation. It is also conceptually appealing that lasing in strong coupling is best realized when strong coupling itself is optimum, i.e., when the inequality (7) is maximum (that is, its rhs is minimum). The wider picture covering both the quantum and classical regimes also reveals different types of thresholds, namely, from quantum ($g^{(2)} < 1$) to classical ($g^{(2)} = 1$) statistics when $\gamma_\sigma < \gamma_a$, and from thermal noise ($g^{(2)} > 1$) to classical statistics, which is the conventional case, when $\gamma_\sigma > \gamma_a$. The intermediate situation where $\gamma_a = \gamma_\sigma$ bridges between Poissonian statistics on both sides. If one would set the criterion for lasing to be the efficiency of growth of the intensity, the negative-jump would yield an "anti-threshold" where stimulated emission spoils the efficiency of cavity population, strong-coupling being more efficient. This jump neatly and fundamentally separates two regions that differ only by the fact that $n_a < 1$ in the former case and $n_a > 1$ in the latter, but are otherwise sharing the same growth of the photon intensity with pumping and Poissonian statistics, that is, both displaying the two main features of a laser. It is therefore adequate to denote them both as lasing. We propose the terminology of "single-photon lasing" and "stimulated-emission lasing" to label the two sides of the quantum regime. The terminology of a "single-photon laser", seemingly contradictory in terms, nevertheless comforts the concept of coherence as the chief characteristic of lasing. This is not a large intensity that characterizes lasing, but the fact that the emitted photons are uncorrelated the ones from the other. We have shown how this definition could be extended down to vanishing intensities of the field, where the very scarce photons emitted retain this property. This is in stark contrast with a natural source where independent events leads to bunched photons,[27] a property which is also independent of the intensity. The same applies to the terminology of "stimulated-emission lasing" which is not a pleonasm in a modern understanding of lasing, where the mechanism is disconnected from its result.

4. UNIVERSAL TRANSITION

The distinction between these two regimes of single-photon and stimulated emission lasing is necessary because although the ideal scenario of thresholdless lasing is indeed realized for the limiting cases which we have discussed, it breaks down in between. Delimited by the linear regime, Eq. (4), and the stimulated emission regime, Eq. (5), lies what we will call the "*quantum regime*", where both the intensity and the statistics deviate from the ideal trend. This is shown in Fig. 1(a), where one can see that the case $\gamma_\sigma = \gamma_a$ accounts for both the linear and the lasing regions with the same line, but with a small deviation in an intermediate region. While this deviation is little apparent in the intensity, it is significant in the statistics of the photon field $g^{(2)}$, shown in Fig. 1(b). In the second order statistics, the passage through the quantum regime is markedly located as a "bump" in an otherwise constant $g^{(2)} = 1$. This disruption of the absence of a jump in the input–output characteristics between the two linear relationships when $C_1 = C_2$ and the fluctuation in the statistics of the field are, in the one-atom laser, linked to the dynamics involving the first few rungs of the Jaynes–Cummings ladder. More particularly, they are linked to the second rung that prevents the formation of an uncorrelated two-photon state, a violation of the symmetry requirement of its wavefunction which can be mimicked with few or many particles.

The most remarkable feature of the transition between these two types of lasing is that it is universal. This follows from the strong coupling limit, where the term featuring κ_σ^{-1} in Eq. (2) becomes negligible, in which case the shape is invariant for the dimensionless parameters P_σ/γ_a and γ_σ/γ_a, for all values of g. It is shown in Fig. 2, along with the physical origin of this fluctuation in statistics, displayed as the difference between the distribution $p(n) = \langle n|\rho|n\rangle$ realized in the system and the ideal Poissonian statistics with the same mean value n_a:

$$\delta_n = p(n) - e^{-n_a} n_a^n / n!\,. \tag{11}$$

In the one-photon lasing region (1–3) in Fig. 2, the system is forced into the lowest rung $n = 1$ of the Jaynes–Cummings ladder, resulting in lower probabilities to have two photons than in an ideal laser of the corresponding intensity n_a. This imbalance for δ_n with $0 \leq n \leq 2$ grows linearly with pumping power and, in the transition region (4–7), it spreads over a wider range of n, with an excess of photons nearby the maximum of the distribution while the neighbouring n-photon states are depleted to compensate. In the stimulated emission lasing region (8), this perturbation in statistics becomes both broader and weaker, allowing the system to recover the exact

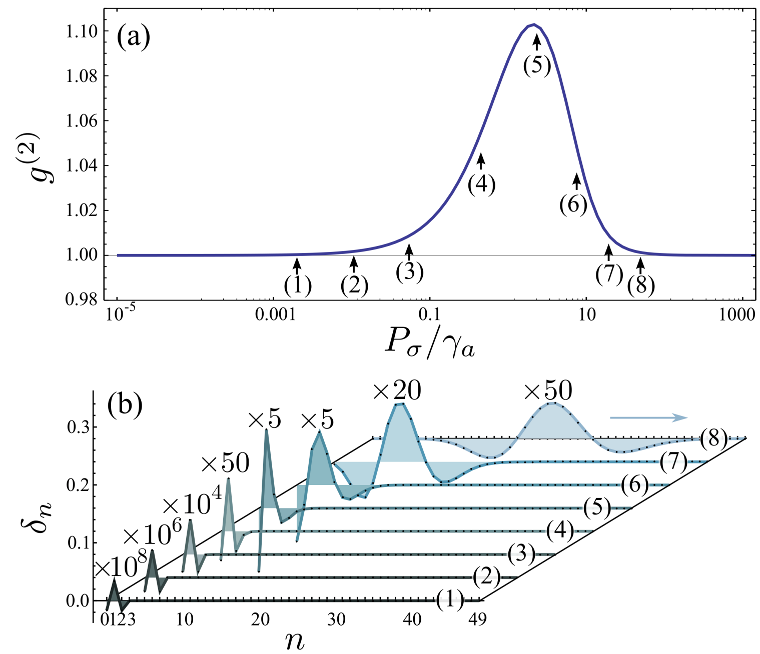

Figure 2. (a) universal curve for $g^{(2)}$ when going from one-photon lasing to stimulated emission lasing, $\gamma_\sigma = \gamma_a$, and (b), deviation of the statistics realized from a Poissonian distribution, Eq. (11), for the points marked by arrows in (a). The maximum value ≈ 1.10282 is the same for any system realizing lasing in strong-coupling. δ_n is magnified by the values shown.

Poissonian fluctuations at high intensities (beyond point (7)). When the system operates too far from the strong coupling regime, the precise deviation from Poissonian statistics becomes non-universal and specific to the system parameters. The shape then deviates from that plotted in Fig. 2(a) and reaches different (lower) values of its maximum. Interestingly, this occurs when the lasing regime established by stimulated emission (after the bump) is no longer reached, that is, no plateau is fully formed where Poissonian statistics is maintained over a range of pumping. We place it at roughly $\gamma_a \approx 0.1g$. This shows that the transition is really a fundamental bridge between the two types of lasing, that disappears if and only if this crossover is not fully realized.

For good enough strong-coupling, universality implies that all systems should exhibit the same maximum value of $g^{(2)}$. Numerically, we estimate these lowest possible values by which the system surpasses Poissonian statistics in the cases of no spontaneous emission and optimum strong coupling to be:

$$g^{(2)} \approx 1.01816\,, \quad \text{at } P_\sigma \approx 4.5989\gamma_a \text{ when } \gamma_\sigma = 0\,,$$
$$g^{(2)} \approx 1.10282\,, \quad \text{at } P_\sigma \approx 2.115\gamma_a \text{ when } \gamma_\sigma = \gamma_a\,.$$

It is difficult to identify a clear operating point that could be defined as the threshold for the single-emitter laser. One can of course make the rather vague statement that it is zero, which does not account well for the variety of situations that can be observed, as discussed above. An unambiguous definition could be the point where $g^{(2)}$ achieves its maximum, now that we have shown this is a universal feature of lasing in strong coupling. In this case, there is no ideal thresholdless laser and the lowest possible threshold is that given by the condition that maximises strong-coupling, $\gamma_\sigma = \gamma_a$, yielding a threshold at a pumping rate slightly larger than twice this common decay rate.

Beyond the two particular limiting cases just outlined of $\gamma_\sigma = 0$ and $\gamma_\sigma = \gamma_a$, there lie all the possible ratio γ_σ/γ_a. From the maximum $g^{(2)}$ obtained, given that it is universal, one can also estimate the pumping rate and the imbalance of the decay rates, quantities otherwise difficult to access directly. Interestingly, such a local maximum of statistics when crossing the thresholds to stimulated emission lasing have been observed in experimental realizations of a few-emitters laser with a shape that resembles our Fig. 2.[28–31] However, in the majority of cases it was linked to an experimental limitation of finite time resolution, whereas it is in our case a manifestation of an intrinsic and universal transition in the system.

5. SUMMARY AND CONCLUSIONS

We have shown that a single-emitter in strong-coupling with a Bose field—typically an atomic-like two-level system in strong-coupling with the photon field—can realize an almost thresholdless laser with, on the one hand, perfect coherence to all orders and, on the other hand, the same and linear increase of the population at all pumping powers. This is realized when the decay rates of the emitter and the Bose field are equal, $\gamma_a = \gamma_\sigma$, in a system in very strong coupling: $\gamma_a, \gamma_\sigma \ll \kappa_\sigma$. Two different mechanisms account for the Poissonian statistics: at low pumping, by maximising strong coupling; at large pumping, by stimulated emission overtaking spontaneous emission. There is a transition between these two regimes with a small deviation from linear increase of the population with pumping and bunching of light. The shape of this transition does not depend on the system parameters, and it therefore acquires a new interest since it allows fundamental tests of the theory at the interface between the quantum and the classical regimes, provide an unambiguous characterization of lasing in strong coupling, quantify the extent of experimental limitations, give a direct access to underlying parameters of the system and set the lowest thresholds achievable in any device relying on strong coupling.

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