

**Métodos matemáticos II.**  
**Examen de recuperación junio 2014**

**I. INTEGRATION (2.5 PTS)**

Calculate:

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2+x^4}.$$

**II. DERIVATION (2.5 PTS)**

Where is the function  $f(z) = \cos(\operatorname{Re}(z)) - i \sinh(\operatorname{Im}(z))$  derivable?

**III. HARMONIC FUNCTION (2.5 PTS)**

For which integers  $n$  is the function  $x^n - y^n$  harmonic?

**IV. DEMONSTRATION (2.5 PTS)**

Provide all the zeros of  $\sin(z)$ , i.e., the numbers  $z \in \mathbf{C}$  such that  $\sin(z) = 0$ . *Prove your result.* (Hint: you can study the modulus of  $\sin(x+iy)$  with  $x$  and  $y$  real numbers, that can be expanded, and use the fact that  $(z=0) \Leftrightarrow (|z|=0)$ ).